Simple Pendulum: Investigating Conditions Affecting Oscillation and Estimating Gravitational Acceleration

Abstract

Simple pendulum experiment involved the examination of the period of oscillation under varying conditions of string length, bob mass, and angle, alongside calculating the acceleration due to gravity. The procedures revealed that for small angles, oscillation period remains relatively constant, with notable changes occurring when the angle exceeds 60 degrees due to the weakening restoring force. Results indicated that mass does not influence the period, as gravitational pull and string tension offset any potential effects. In contrast, significant changes in the period were observed as the string length varied, confirming its direct relationship with oscillation time. The experiment yielded an experimental value for gravitational acceleration of 9. 87 m/s^2, closely aligning with the theoretical value of 9. 82 m/s^2, resulting in a minimal error rate. Possible sources of error were identified: reaction time and measurement inaccuracies. Overall, the findings were deemed valid as less error occurred.

. Introduction

Gravity being one of the most fundamental concepts in physics can be measured through various methods. Equations such as free fall, spring scale, drop tower, and pendulum method can be used to determine the acceleration due to gravity on Earth. The value of gravity on Earth will be calculated through utilizing the simple pendulum experiment where the pendulum's period and length is related to a gravitational constant of 9.81m/s^2. A pendulum is a device that swings back and forth in regular patterns until it runs out of force and consists of a weight called a bob connected to a string. The oscillation can be affected by the mass of the bob, angle, length of string, and location where the experiment took place but for smaller pendulum experiments can be negligible. Since gravity plays an important role in the pendulum's motion, it is the force that acts and pulls the bob downward and towards the center of Earth. Furthermore, restoring force acting on the object usually returns it to equilibrium position. With regards to the equation of a simple pendulum, the restoring force perpendicular to the string is negative and through manipulation the equation for simple harmonic motion to period can be achieved. The equation for period uses the law of isochronism where the length of string affects the period of oscillation (Yuningsih, Sardjito, Dewi,, 2020). The dependence of the period of oscillation to the length of the string, mass of the bob, and angle were further investigated along with the best estimate of acceleration due to gravity using a pendulum.

I. Methodology

Learners followed the first procedure of gathering the materials such as iron stand, pendulum bob, inelastic string, stop watch, and pendulum clamp. Some members measured the length of the string being 37.5 cm while others prepared the entire setup where they attached the pendulum clamp to the iron stand.

Figure 1: Set-up

The string was then tied to both the bob weighing 100 grams and pendulum clamp before it was oscillated at angles 5 °, 30 °, 45 °, and 60 °. Next, time was recorded after completing 20 periods in Table 1 for each three trials. Period of pendulum was found through dividing average time by 20 which was recorded in experimental period Table 1. Theoretical period was determined using equation 1.

$$T_o = 2\pi\sqrt{\frac{L}{g}} \quad (1)$$

Pendulum length was set to 30 cm as the bob oscillates at 10 ° with respect to the resting equilibrium. Time was measured after the pendulum completed 20 periods and was recorded in Table 2 for each four different masses. The procedure for calculating experimental and theoretical periods was similar to the first one.

With the pendulum bob having a mass of 100 grams, the pendulum oscillated at 5 ° with respect to the resting equilibrium position for each different string lengths. The same procedure of calculating experimental and theoretical period was done and it was recorded along with time in Table 3.

From the data obtained in Table 3, T^2 versus L were graphed. Average time was raised to two and it was graphed in y-axis while length used the x-axis. The values were switched in accordance to the fact that independent variables should represent x and y as the outcome or dependent variables.

Linear regression of y = 3.91x + 0.019 was achieved and experimental estimates of acceleration due to gravity were computed through manipulating equation 1 in the form of y = mx + b as shown in equation 2-4.

$$T\text{^}2 = 3.910\,L + 0.019 \quad (2)$$

$$T^2 = 4\pi^2\frac{L}{g} \quad (3)$$

$$T^2 = \frac{4\pi^2}{g}L + 0 \quad (4)$$

Since T^2 and L is equal to the slope which was rounded to 4, equation 4 was manipulated to arrive at the equation of gravity with a value of 9.87 m/s^2.

$$g = \frac{4\pi^2}{4} \quad (5)$$

Theoretical period was computed through manipulation of equation 1 to yield gravity as shown through equation 6.

$$g = 4\pi^2\frac{L}{T^2} \quad (6)$$

Using Table 3, different lengths were computed alongside their respective time period which yielded with specific gravitational acceleration therefore, the answers were averaged. Plugging the respective values of length and time raised to two will result in an average of 9.82 m/s^2.

III. Results and Discussion

Table 1. Periods for varying angles

Angle °	20 T (s)			Ave, T(s)	Period, T (s)		Error (%)
	Trial 1	Trial 2	Trial 3		Experimental	Theoretical	
5 °	24.28	24.02	24.41	24.24	1.21		0.82

30 °	24.32	24.22	24.31	24.28	1.21	1.22	0.82
45 °	24.87	24.63	24.87	24.79	1.22		0
60 °	25.16	25.15	25.13	25.15	1.26		3.28

On the first procedure of turning the angle into a dependent variable, the mass chosen was 100 grams and string length is 37.5 cm. The results from the angles are almost similar, highlighting the idea that it is only for small angles that time recorded during its’ 20 times oscillation is not affected. It can be noted that once an angle reaches a certain threshold or in the experiment 60 degrees, a slight change in recorded time occurred wherein it increased by one. This is due to the restoring force which is proportional to the angle of displacement. Since it's the largest angular amplitude, the pendulum swings farther and the restoring force which pushes the pendulum to swing becomes weaker resulting in slower acceleration and leading to more time expended. Overall, the findings support the first law of isochronism by Galileo which states the pendulum is independent of amplitude so long as it is sufficiently small.

Table 2. Periods for varying mass

Mass, m (g)	20 T (s)			Ave, T(s)	Period, T (s)		Error (%)
	Trial 1	Trial 2	Trial 3		Experimental	Theoretical	
10	21.04	20.95	21.20	21.06	1.05	1.10	4.55
20	21.34	21.34	21.51	21.40	1.07		2.73
50	21.04	21.27	21.30	21.20	1.06		3.64
100	21.92	21.58	22.14	21.88	1.09		0.90

Moving on to procedure two where the mass of the pendulum Bob becomes the dependent variable, the masses are 10 grams, 20 grams, 50 grams, and 100 grams. No large changes in time recorded as every value is almost similar suggesting that mass doesn't affect the period of a simple pendulum. This can be said with great confidence as mass was not a factor in calculating the time period. This is also due to gravity pulling the Bob downward while the tension of the string stabilizes this force as it moves. The force and inertia of the Bob is proportional to mass thus mass does not affect the motion no matter how small or large it is. With large mass comes great inertia and requires more acceleration so it responds to gravity similar to a lighter mass. The findings correspond with the law of mass wherein the time period does not rely on the mass of the pendulum bob.

Table 3. Periods for varying lengths

Length, L (cm)	20 T (s)			Ave, T(s)	Period T (s)		Error (%)
	Trial 1	Trial 2	Trial 3		Experimental	Theoretical	
10	11.89	12.03	11.85	11.92	0.60	0.63	4.76
20	17.18	17.10	17.66	17.31	0.87	0.90	3.33
30	21.67	21.85	21.89	21.80	1.09	1.10	0.90
37.5	24.28	24.02	21.41	24.24	1.21	1.22	0.82

Third procedure is the length of the string becoming the dependent variable and notable changes in time period occurred for each length. The second law or law of length states that the period of oscillation is directly proportional to the square root of length. Since the length of the pendulum is directly related to the time period: the longer the length, the slower the swing resulting in longer time. Inversely, a shorter pendulum length will have faster oscillation within a lesser time period. Moreover, a pendulum with a shorter string has higher frequency resulting in rapid swing.

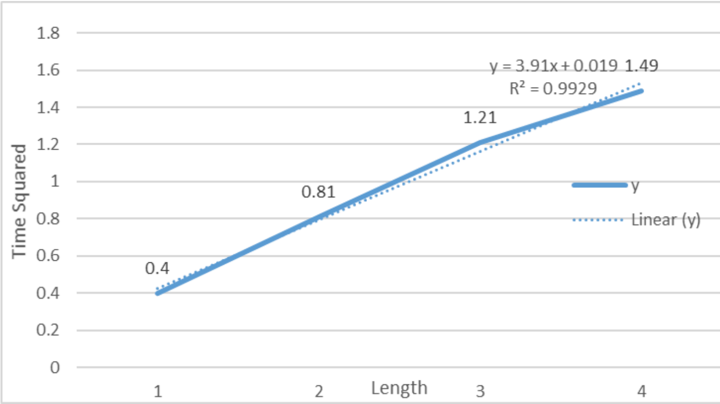

Figure 2: Linear Regression

Theoretical value was solved through manipulation of the original time period formula to have g the unknown value and this bore 9.82 m/s^2. Experimental value was found to be 9.87 m/s^2 through graphing the values of T^2 as y and L as x. The equation was manipulated further to be in the form y = mx + b as seen in equation 2. By using linear regression, the slope was given by excel to be 3.91 which is equivalent to $4\pi^2$over g. Equating those two and solving for the unknown g is how the gravitational acceleration was calculated. Experimental gravitational acceleration value is the same as Lowrie which is 9.8 m/s^2 and differs by 0.51% error compared to the theoretical gravitational acceleration value of 9.82 m/s^2.

Rate of errors are within minimal levels, making the findings valid. Certain sources of error are undoubtedly present. The first possible source is reaction time during stopwatch recording where hand-and-eye coordination is put to the test. Second is the way the angle is measured during the experiment where again, eyesight is put to the test along with how stable or a person does not experience essential tremors or slight hand shaking during performing tasks.

IV. Conclusions

In a nutshell, this experiment determined if conditions such as string length, bob mass, and angle affects the oscillation period of a pendulum, alongside computing the acceleration due to gravity. Significant changes in the period were observed when the length of the string varies, signifying it has a direct relationship with oscillation time. In contrast, mass has no influence in the period of a simple pendulum because gravitational pull and string tension offset any potential changes. Lighter and heavier mass swings the same given they have the same string length and angular amplitude. The procedures for small angles revealed that the oscillation period remains constant for smaller angles. The findings support the law of a simple pendulum: law of length, law of mass, and law of isochronism. The gravitational acceleration The experiment yielded an experimental value of 9.87 m/s^2 for experimental and 9.82 m/s^2 theoretical. Possible sources of error are human reaction time, essential tremors alongside measurement inaccuracies’.

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VI. References

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